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Spin-wave and spin-fluctuation contributions to the magnetoresistance of weak itinerant-electron ferromagnets

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Abstract

The spin-wave and exchange-enhanced spin-density fluctuation contributions to the electrical resistivity, $\rho(T)$, of weak itinerant-electron ferromagnets are calculated in the *absence* and *presence* of the magnetic field, H, employing the two-band (s- and d-band) model and the version of spin-fluctuation theory for d-band that makes use of the Ginzburg–Landau formalism. These *self-consistent* calculations (i) mark a substantial improvement over the previous theoretical treatments in that they completely dispense with the *unrealistic* electron-gas approximation, (ii) account for the effect of H on the contributions to $\rho(T)$ arising from spin waves (spin-density fluctuations) at low temperatures (at intermediate temperatures and for temperatures close to the Curie point, T_C), and (iii) regardless of the nature of the low-lying magnetic excitations that dominantly contribute to the *negative* magnetoresistance, $\Delta\rho/\rho$, in different temperature ranges, yield analytical expressions that basically have a simple general form $(\Delta\rho/\rho) = 1 - [\rho(T, H)/\rho(T, H = 0)] \cong aH - bH^2$. The expressions, so obtained, not only permit a quantitative determination of the suppression of spin waves and spin fluctuations by the magnetic field, H, in weakly ferromagnetic metals from the magnetoresistance data but are also capable of yielding useful information about the band structure.

1. Introduction

The theoretical formalisms [1–11] that make an adequate provision for the collective nature of electron–hole pair excitations (i.e., *non-propagating* spin fluctuations) go far beyond the mean-field treatment of the Stoner–Wohlfarth model [12] in that they provide straightforward explanations for the attributes of weak itinerant-electron (WI) ferromagnets that the Stoner–Wohlfarth model failed to account for (for details and a comprehensive reference to the relevant literature, see [13]). Most of these theories deal with the spin-fluctuation contribution to various thermodynamic properties of WI ferromagnets and only a few of them [1, 6, 8, 11] address the important issue of the suppression of spin fluctuations by the external magnetic field (H_{ext}). The latter class of theories [1, 6, 8] makes use of the electron gas model to calculate a field- and temperature-dependent static susceptibility that is *consistent* with the one deduced from the magnetic equation of state. Such an approach cannot be regarded as satisfactory for the following reasons. First, the electron gas model forms an *oversimplified* description of the band structure of *real* WI magnetic systems. Second, a number of adjustable parameters [1, 6, 8] have been used to arrive at a quantitative agreement with the experiment. Third, the sole theoretical attempt [6] to calculate the magnetoresistance of weakly and nearly ferromagnetic metals completely neglects the spin-wave contribution (which almost *completely accounts for* [14] the temperature dependence of magnetization observed in Ni_3Al at low temperatures and whose overwhelming presence in Ni_3Al is well documented [15] from neutron scattering experiments) and exclusively deals with the spin-fluctuation contribution only at low temperatures.

To partly remedy this situation, the present author has recently reported [13] the results of an explicit *self-consistent* calculation of the zero-point and thermally excited spin-fluctuation contributions to magnetization in WI ferromagnets in the presence and absence of an external magnetic field, based on the version of spin-fluctuation theory that makes use of the Ginzburg–Landau formalism. These results revealed the following.

(i) The zero-point spin fluctuations have a *major* share in renormalizing the Landau coefficients of the Stoner theory, are essentially *insensitive* to H_{ext}, and make a small but significant contribution to the temperature dependence of magnetization in WI ferromagnets, particularly at intermediate temperatures.
(ii) By contrast, the thermally excited spin fluctuations (*propagating* transverse spin fluctuations, i.e., spin waves, at low temperatures and *non-propagating* longitudinal as well as transverse spin fluctuations at intermediate temperatures and for temperatures close to the Curie point, T_C) almost *entirely* account for the dependences of magnetization on temperature and field, and get *strongly suppressed* by H_{ext}.
(iii) The suppression of thermally excited fluctuations by H_{ext} for temperatures just outside the critical region but below T_C should follow the $(H_{ext})^{1/2}$ power law.

Subsequently, these theoretical predictions have been validated [14, 16] by the results of high-resolution magnetization measurements on the WI ferromagnets Ni_3Al and $Ni_{75-x}Fe_xAl_{25}$. The success of the above theoretical approach [13] in correctly describing the experimental results on magnetization has motivated the author to extend this formalism to calculate the spin-wave and spin-fluctuation contributions to magnetoresistance in WI ferromagnets.

In the following section, the two-band (s- and d-band) model and the version of spin-fluctuation theory for the d-band electrons that is based on the Ginzburg–Landau formalism are used to calculate the contributions to the electrical resistivity, $\rho(T)$, of weak itinerant-electron (WI) ferromagnets arising from the *propagating* transverse spin fluctuations (spin waves) at low temperatures and *correlated* electron–hole pair *collective* excitations (exchange-enhanced *non-propagating* longitudinal and transverse spin-density fluctuations) at intermediate temperatures and for temperatures close to the Curie point, T_C, in the *absence* and *presence* of the magnetic field, H. Although the starting point (equation (1) of section 2) of the present *self-consistent* calculations is the same as that of the previous ones [5, 6, 17], these calculations go far beyond the earlier theoretical approaches [1, 5, 6, 8] in completely *dispensing* with the electron-gas approximation and in accounting for the effect of magnetic field on the contributions to resistivity arising from spin waves (SWs) at low temperatures and spin-density fluctuations (SFs) at intermediate temperatures and for temperatures close to T_C. Another important feature of these calculations is that they yield expressions for the SW and SF contributions to *negative* magnetoresistance, $\Delta\rho/\rho$, in WI ferromagnets that permit a quantitative determination of the suppression of spin waves and spin-density fluctuations by the magnetic field, H, in weakly ferromagnetic metals from the magnetoresistance data and have the potential of yielding useful information about the band structure. In the appendix, the classical approximation is used in conjunction with the temperature- and field-dependent spin-wave cut-off wavevector to calculate the spin-wave contributions to *negative* magnetoresistance and the 'in-field' magnetization of WI ferromagnets. The resulting expressions not only reproduce the experimentally observed $H^{1/2}$ law for the suppression of spin waves by H, as evidenced in the 'in-field' magnetization at low temperatures and moderate fields, in such systems but also the field variations of the magnetoresistance predicted by more accurate theoretical expressions derived in the following section. The $H^{1/2}$ law is already well established in the case of localized-moment ferromagnets.

2. Electrical resistivity and magnetoresistance

Within the framework of the two-band (s- and d-band) model, a standard theoretical treatment [6, 17, 18] of the scattering of conduction (s) electrons by the spin-density fluctuations of the d-electrons via the s–d exchange interaction yields the following expression for the electrical resistivity:

$$\rho(T) = \frac{3}{4}\left(\frac{m}{ne^2}\right)\Im^2_{\text{s-d}}\left(\frac{E^{\text{s}}_{\text{F}}}{\hbar}\right)N(E^{\text{s}}_{\text{F}})N(E^{d}_{\text{F}})\left(\frac{k^{d}_{\text{F}}}{k^{\text{s}}_{\text{F}}}\right)^4\left(\frac{E^{d}_{\text{F}}}{E^{\text{s}}_{\text{F}}}\right)r(T) \tag{1}$$

with

$$r(T) = \frac{1}{T}\int_0^{2k^{\text{s}}_{\text{F}}/k^{d}_{\text{F}}} \mathrm{d}q\, q^3 \int_{-\infty}^{\infty} \mathrm{d}\omega\, \omega n(\omega)\,[n(\omega)+1]\,\mathrm{Im}\chi_r(q,\omega) \tag{2}$$

$$n(\omega) = \left[\exp(\omega/T) - 1\right]^{-1} \tag{3}$$

$$\chi_r(q,\omega) = \left[\chi_\parallel(q,\omega) + 2\chi_\perp(q,\omega)\right]/3 \tag{4}$$

$$\mathrm{Im}\chi_\nu(q,\omega) = \omega\chi_\nu(q)\frac{\Gamma_\nu(q)}{\omega^2 + \Gamma^2_\nu(q)} \tag{5}$$

$$\chi_\nu(q) = \chi_\nu(q,\omega=0) = \chi_\nu(0)\frac{\kappa^2_\nu}{\kappa^2_\nu + q^2} \tag{6}$$

$$\Gamma_\nu(q) = \gamma_\nu q \chi^{-1}_\nu(q) = \Gamma_\nu q(\kappa^2_\nu + q^2) \tag{7}$$

$$\chi_\nu(0) = \chi_\nu(q=0) = (c_\nu\kappa^2_\nu)^{-1} \tag{8}$$

$$\Gamma_\nu = c_\nu\gamma_\nu. \tag{9}$$

In the above expressions, m and n are the effective mass and number density of the s-electrons, respectively, $E^{\text{s}}_{\text{F}}(E^{d}_{\text{F}})$ is the Fermi energy of the s(d)-band measured from the bottom of the band, $k^{\text{s}}_{\text{F}}(k^{d}_{\text{F}})$ and $N(E^{\text{s}}_{\text{F}})(N(E^{d}_{\text{F}}))$ respectively are the Fermi wavevector and the density of states at the Fermi energy per spin per atom of the s(d)-band, $\Im_{\text{s-d}}$ is the s–d exchange coupling constant, $n(\omega)$ is the Bose function, $\chi_\parallel(q,\omega)$ and $\chi_\perp(q,\omega)$ are the *dynamical wavevector-dependent* longitudinal and transverse susceptibilities [10, 13], $\Gamma_\nu(q)$ is the relaxation frequency of a spontaneous spin fluctuation of wavevector q and polarization ν ($\nu \equiv \parallel$ or $\perp$) [10, 13], $\chi_\nu(0) \equiv \chi_\nu(T,H)$ is the field- and temperature-dependent susceptibility, c_ν is the coefficient of the gradient term in the Ginzburg–Landau expansion [10], and the quantity γ_ν depends on the shape of the density of states (DOS) curve near E^{d}_{F} [10]. The temperature dependence of resistivity is completely accounted for by the dimensionless quantity $r(T)$, which, for the sake of convenience, is expressed in reduced units $\omega \equiv \hbar\omega/E^{d}_{\text{F}}$, $T \equiv k_{\text{B}}T/E^{d}_{\text{F}}$, $q \equiv q/k^{d}_{\text{F}}$,

$\kappa_\nu \equiv \kappa_\nu / k_F^d$, $\Gamma_\nu(q) \equiv \hbar\Gamma_\nu(q)/E_F^d$, $c_\nu \equiv c_\nu (k_F^d)^2$, $\gamma_\nu \equiv \hbar\gamma_\nu k_F^d/E_F^d$, and $\Gamma_\nu \equiv \hbar c_\nu \gamma_\nu (k_F^d)^3/E_F^d$. Note that $\chi_\parallel(q,\omega) = \chi_\perp(q,\omega)$ in the paramagnetic phase (i.e., for $T > T_C$) but the dynamical wavevector-dependent longitudinal and transverse susceptibilities are *not equal* in the ferromagnetic phase (i.e., for $T > T_C$).

2.1. *Thermally excited spin fluctuations*

2.1.1. Low temperatures. Theoretical calculations [1] of the energy dispersion, $E(q)$, of magnetic excitations in a weakly ferromagnetic metal reveal the following. At $T = 0$,

(i) spin-wave excitations, representing the bound states for electron–hole pairs, are confined to a small region around $q = 0$ in the Brillouin zone; for values of q close to zero, these states have lower energy and are separated by an energy gap from the energy continuum, corresponding to the Stoner single-particle spin-flip excitations.

(ii) This energy gap reduces as q increases so much so that beyond a certain threshold value of $q = q_0$, the spin-wave dispersion curve enters the Stoner excitation continuum with the result that *propagating transverse* spin fluctuations (spin waves) get damped.

For $q > q_0$, the magnetic excitations in the continuum are the overdamped (*non-propagating*) modes of exchange-enhanced longitudinal and transverse spin-density fluctuations. Since spin-wave modes of larger and larger q are excited as the temperature is raised from $T = 0$, the transition at $q = q_0$ from well-defined spin waves to *non-propagating* exchange-enhanced *transverse* spin fluctuations manifests itself at a certain finite value of temperature in the measurement of thermodynamic quantities such as magnetization, electrical/thermal resistivity, and specific heat. By contrast, the thermally excited *non-propagating* exchange-enhanced *longitudinal* spin-density fluctuations persist down to $q = 0$ and coexist with, but are swamped by, spin waves for $q \leqslant q_0$.

At low temperatures ($T \ll T_C$), the main contribution to $\rho(T)$ arises from long-wavelength ($q \leqslant q_0$) low-frequency spin-wave (SW) modes. This contribution is obtained by inserting the following expression [10, 13] for $\mathrm{Im}\chi_r(q,\omega)$ in equation (2) and then evaluating the integrals:

$$\mathrm{Im}\chi_r(q,\omega) = 2\,\mathrm{Im}\chi_\perp(q,\omega)/3 \tag{10}$$

with

$$\mathrm{Im}\chi_\perp(q,\omega) = (\pi/2)\omega\chi_\perp(q)\,[\delta(\omega-\omega(q)) + \delta(\omega+\omega(q))] \tag{11}$$

and the spin-wave propagation frequency $\omega(q)$ given by [10]

$$\begin{aligned}\hbar\omega(q) &= g\mu_B M(T,H)\chi_\perp^{-1}(q) = g\mu_B M(T,H)(\chi_\perp^{-1}(0) + c_\perp q^2 + \cdots)\\ &= g\mu_B H + D(T,H)q^2 + \cdots\end{aligned} \tag{12}$$

or, in reduced units, by

$$\omega(q) = m_r\chi_\perp^{-1}(q) = h_r + d_{SW}q^2 + \cdots \tag{13}$$

where $m_r = g\mu_B M/E_F^d$, $h_r = g\mu_B H/E_F^d$, $d_{SW} = D(k_F^d)^2/E_F^d = m_r c_\perp$, $c_\perp \equiv c_\perp (k_F^d)^2$, $\chi_\perp^{-1} = H/M(T,H)$, the *effective* field H is the external magnetic field, H_{ext}, corrected for the demagnetizing field, H_{dem}, and other anisotropy fields, H_A, i.e.,

$$H = H_{ext} - H_{dem} + H_A = H_{ext} - 4\pi N M(T, H_{ext}) + H_A,$$

N is the demagnetizing factor, g is the Landé splitting factor, and $D(T,H) = g\mu_B M(T,H)c_\perp$ is the spin-wave stiffness. Combining equations (2), (3), (10), (11) and (13) yields the result

$$r(T,h_r) = \left(\frac{2\pi m_r}{3T}\right)\int_0^{q_0} \mathrm{d}q\, q^3 \frac{\omega(q)\mathrm{e}^{\omega(q)/T}}{(\mathrm{e}^{\omega(q)/T}-1)^2} \tag{14}$$

where the upper limit $2k_{\mathrm{F}}^{\mathrm{s}}/k_{\mathrm{F}}^{d}$ for the integral over q has been replaced by the upper cut-off wavevector q_0 for the spin-wave modes. In the absence of the external magnetic field (i.e., when $H = h_r = 0$), the spin-wave excitations are confined to a very narrow region around $q = 0$ of the Brillouin zone in WI ferromagnets so that the upper limit q_0 can be taken to be ∞ without sacrificing accuracy. When $H = h_r = 0$, $\omega(q) = d_{\mathrm{SW}}q^2$ and the definite integral in equation (14) can be solved, using the standard result

$$\int_0^{\infty} \mathrm{d}x\, x^n \mathrm{e}^x (\mathrm{e}^x - 1)^{-2} = \Gamma(n+1)\zeta(n),$$

to finally arrive at the following expression for the spin-wave contribution to the zero-field resistivity in *usual units*:

$$\rho_{\mathrm{SW}}(T, H = 0) = \frac{\pi}{3}\rho_0 \Gamma(3)\zeta(2)\left(\frac{g\mu_{\mathrm{B}}M}{\hbar}\right)\left(\frac{k_{\mathrm{B}}T}{D}\right)^2 \quad (15)$$

with

$$\rho_0 = \frac{3}{4}\left(\frac{m}{ne^2}\right)\Im_{\mathrm{s-d}}^2 N(E_{\mathrm{F}}^{d})N(E_{\mathrm{F}}^{\mathrm{s}})(k_{\mathrm{F}}^{\mathrm{s}})^{-4}. \quad (16)$$

In the *presence* of the magnetic field (H), the spin-wave excitations get progressively suppressed by the field because H causes a gap in the spin-wave spectrum in accordance with equation (12), and *increases* the spin-wave cut-off wavevector q_0. Consequently, equation (14) assumes the form

$$r(T, h_r) = \frac{\pi}{3}m_r\left(\frac{T}{d_{\mathrm{SW}}}\right)^2\left[\int_{\omega(0)/T}^{\omega(q_0)/T}\mathrm{d}x\,\frac{x^2\mathrm{e}^x}{(\mathrm{e}^x-1)^2} - \left(\frac{h_r}{T}\right)\int_{\omega(0)/T}^{\omega(q_0)/T}\mathrm{d}x\,\frac{x\mathrm{e}^x}{(\mathrm{e}^x-1)^2}\right] \quad (17)$$

where $\omega(q_0) = \omega(q = q_0) = h_r + d_{\mathrm{SW}}q_0^2$ and $\omega(0) = \omega(q = 0) = h_r$. If the very weak dependence of q_0 on H is neglected and the upper limit of the integrals appearing in equation (17) is replaced by ∞ (i.e., set $\omega(q_0)/T = \infty$), the final expressions for the spin-wave contribution to the *negative* magnetoresistance, $(\Delta\rho/\rho)_{\mathrm{SW}}$, at low temperatures are given by

$$\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{SW}} = 1 - \frac{\rho_{\mathrm{SW}}(T,H)}{\rho_{\mathrm{SW}}(T,H=0)} = -[\Gamma(3)\zeta(2)]^{-1}\left[h\ln(\mathrm{e}^h-1) + 2\sum_{n=1}^{\infty}\frac{(-1)^n(\mathrm{e}^h-1)^n}{n^2}\right] \quad (18)$$

for $(\mathrm{e}^h - 1) < 1$, or equivalently, for $h < 0.693$ and

$$\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{SW}} = 1 - \frac{\rho_{\mathrm{SW}}(T,H)}{\rho_{\mathrm{SW}}(T,H=0)} = -[\Gamma(3)\zeta(2)]^{-1}\left[h\ln(\mathrm{e}^h-1) - \left\{\ln(\mathrm{e}^h-1)\right\}^2 - 2\sum_{n=1}^{\infty}\frac{(-1)^n(\mathrm{e}^h-1)^{-n}}{n^2}\right] \quad (19)$$

for $(\mathrm{e}^h - 1) > 1$ or $h > 0.693$. Equations (18) and (19) quantify the suppression of spin waves by field, and the consequent change in $(\Delta\rho/\rho)_{\mathrm{SW}}$ with field, at low and intermediate fields ($h < 0.693$), and at high fields ($h > 0.693$), respectively. Since most of the suppression occurs at fields $h < 0.693$, equation (18) is more relevant to the present study, which mainly focuses on the suppression of low-lying magnetic excitations by field at moderate fields. The variations of $(\Delta\rho/\rho)_{\mathrm{SW}}$ with field at different temperatures in the low-temperature region, predicted by equation (18), are depicted by the continuous curves in figure 1. In order to make these variations more transparent, the $(\mathrm{e}^h - 1)$ term, appearing in equation (18), is *approximated* by h for $h \ll 1$ and only the first two (leading) terms in the sum over n are retained, with the result that

$$\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{SW}} \cong 0.304\left[-h\ln h + 2h - \frac{1}{2}h^2\right]. \quad (20)$$

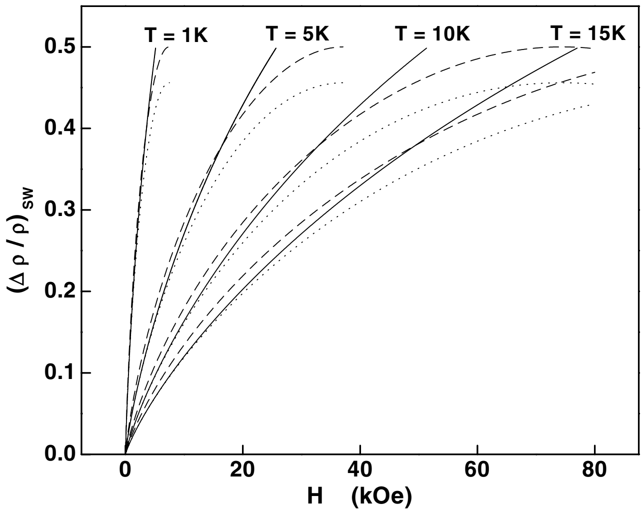

Figure 1. The theoretical variations of the spin-wave contribution to negative magnetoresistance, $(\Delta\rho/\rho)_{SW}$, with magnetic field at different temperatures in the low-temperature region predicted by equations (18) (continuous curves) and (20) (dashed curves) of the text as well as by equation (A.6) (dotted curves) of the appendix. Note that to facilitate a direct comparison between the two types of variations, the $(\Delta\rho/\rho)_{SW}$ data computed from equation (A.6) have been multiplied by 0.5.

The theoretical curves, based on equations (18) (continuous curves) and (20) (dashed curves) shown in figure 1, reveal the following.

(i) For a given field strength, the suppression of spin waves by the magnetic field, as reflected in $(\Delta\rho/\rho)_{SW}$, *decreases* with *increasing* temperature.

(ii) At a given temperature, the suppression of spin waves at *low* and *intermediate* fields is predominantly due to the first $(-h\ln h)$ and second $(2h)$ terms in equation (20), respectively; the tendency of $(\Delta\rho/\rho)_{SW}$ to saturate at high fields is a consequence of the competition between the third term and the first two terms in equation (20).

Note that equation (18) is not valid when $h > 0.693$ and that the limiting value (0.5) of $(\Delta\rho/\rho)_{SW}$ is reached when the reduced field approaches its highest allowed value of $h = 0.693$. An alternative approach to arrive at a result (equation (A.6)) equivalent to equation (18) employs the classical approximation in conjunction with the temperature- and field-dependent spin-wave cut-off wavevector, as elucidated in the appendix. The field variations of $(\Delta\rho/\rho)_{SW}$ at different temperatures predicted by equation (A.6) (represented by the dotted curves) are compared with those yielded by equation (18) in figure 1. Compared to equation (18), equation (A.6) overestimates the magnitude of $(\Delta\rho/\rho)_{SW}$ at any given value of H by nearly a factor of *two*. The field variations predicted by equation (18), or even equation (20), are in better accord with the experiment [21, 22]. The formalism leading to equation (A.6), when used to derive the spin-wave contribution to the 'in-field' magnetization (for details, see the appendix), yields the $H^{1/2}$ law for the suppression of spin waves by H at low and moderate fields for weak itinerant-electron ferromagnets. This prediction is in agreement with the experimental observations [14, 16, 19]. Interestingly, the same result (i.e., the $H^{1/2}$ law) was derived long ago by Holstein and Primakoff [20] in the case of localized-moment ferromagnets.

2.1.2. Low and intermediate temperatures. At intermediate temperatures but still well below T_C, the spin-fluctuation contribution to the magnetoresistance becomes more important than the spin-wave contribution. For temperatures below T_C, the longitudinal and transverse spin-fluctuation contributions to the resistivity, and hence to the magnetoresistance, have to be treated differently since the imaginary part of the dynamical wavevector-dependent susceptibility has different lower bounds ($q_\perp = q_0, \omega = 0$) and ($q_\parallel = 0, \omega = 0$) in the (q, ω)-plane for the transverse and longitudinal fluctuations (as already elucidated in the opening paragraph under section 2.1.1). From equations (2)–(5), the spin-fluctuation contribution is given by

$$\begin{aligned} r_\nu(T, h_r) &= \left(\frac{2\gamma_\nu}{T}\right) \int_{q_\nu}^{q_c} \mathrm{d}q\, q^4 \int_0^\infty \mathrm{d}\omega\, \frac{\mathrm{e}^{\omega/T}}{(\mathrm{e}^{\omega/T} - 1)^2} \frac{\omega^2}{\omega^2 + \Gamma_\nu^2(q)} \\ &= \gamma_\nu \int_{q_\nu}^{q_c} \mathrm{d}q\, q^4 \left[-1 - \frac{1}{2Z} + Z\psi'(Z)\right] \end{aligned} \tag{21}$$

with $q_c \equiv 2k_F^s/k_F^d$, $Z = \Gamma_\nu(q)/2\pi T$ and $q_\nu = q_\parallel$ or $q_\perp$; the integral over ω has been solved using the standard result

$$\int_0^\infty \mathrm{d}x\, \frac{\mathrm{e}^x}{(\mathrm{e}^x - 1)^2} \frac{x^2}{x^2 + (2\pi Z)^2} = \frac{1}{2}\left[-1 - \frac{1}{2Z} + Z\psi'(Z)\right]$$

where $\psi'(Z)$ is the trigamma function. At low and intermediate temperatures, Z is large and the function $\psi'(Z)$ can be expanded in powers of $(1/Z)$ with the result

$$\psi'(Z) \cong (1/Z) + (1/2Z^2) + (1/6Z^3) - (1/30Z^5) + \cdots.$$

This series converges quickly for large Z and justifies retaining terms in this expansion up to Z^{-3} only, so that

$$\left[-1 - (1/2Z) + Z\psi'(Z)\right] \cong (1/6Z^2)$$

and equation (21) reduces to

$$r_\nu(T, h_r) = \frac{2\pi^2}{3}\left(\frac{\gamma_\nu}{\Gamma_\nu^2}\right) T^2 \int_{q_\nu}^{q_c} \frac{q^2\, \mathrm{d}q}{(\kappa_\nu^2 + q^2)^2}. \tag{22}$$

Now that the spin fluctuations occupy only a small region of the Brillouin zone around $q_\perp = q_0$ or $q_\parallel = 0$ at low and intermediate temperatures, the upper limit of the integration over q can be taken to be $q_c \cong \infty$ without a significant loss of accuracy. Solving the integral in equation (22) and expressing all the quantities in the *normal units* leads finally to the result

$$\rho_\perp(T, H) = \frac{2\pi^2}{9}\gamma_\perp \left(\frac{\rho_0}{q_0}\right)\left(\frac{k_B T}{\hbar\Gamma_\perp}\right)^2 \left\{\left(\frac{q_0}{\kappa_\perp}\right)\left[\frac{\pi}{2} - \tan^{-1}\left(\frac{q_0}{\kappa_\perp}\right) + \frac{(q_0/\kappa_\perp)}{1 + (q_0/\kappa_\perp)^2}\right]\right\} \tag{23}$$

and

$$\rho_\parallel(T, H) = \frac{\pi^3}{18}\gamma_\parallel \left(\frac{\rho_0}{q_0}\right)\left(\frac{k_B T}{\hbar\Gamma_\parallel}\right)^2 \left\{\frac{q_0}{\kappa_\parallel}\right\}. \tag{24}$$

In the above expressions, the field dependence basically originates from the inverse spin correlation length κ_ν (where the subscript ν denotes the transverse $\perp$ or longitudinal $\parallel$ case) which is related to the temperature- and field-dependent transverse (longitudinal) susceptibility $\chi_\perp(0) = M/H$ ($\chi_\parallel(0) = \partial M/\partial H$) as $\kappa_\nu^2 = [c_\nu \chi_\nu(0)]^{-1}$, where $M \equiv M(T, H)$. Note that this relation is an alternative form of equation (8). To calculate $\chi_\parallel(0)$ and $\chi_\perp(0)$, the following *self-consistent* approach has been adopted. We start with the magnetic equation of state [10, 13]

$$H = AM(T, H) + b[M(T, H)]^3 \tag{25}$$

with

$$A = a(T) + b(3\langle m_{\parallel}^2\rangle + 2\langle m_{\perp}^2\rangle), \tag{26}$$

which, for $H = 0$ and $T < T_C$, yields the spontaneous magnetization, $M_0 \equiv M(T, H = 0)$, as

$$M_0 = (-A(T)/b)^{1/2}. \tag{27}$$

For $T < T_C$, A *is negative* and $M(0)$ is finite. In equation (26), $\langle m_{\parallel}^2\rangle$ and $\langle m_{\perp}^2\rangle$ are the thermal variances of the local magnetization parallel and perpendicular to the average magnetization M, respectively, and the detailed expressions for the coefficients $a(T)$ and b are given in [13]. From the magnetic equation of state (MES), equation (25), it immediately follows that

$$\chi_{\perp}^{-1}(T, H) = \frac{H}{M} = A + bM^2 = b(M^2 - M_0^2) \tag{28}$$

$$\chi_{\perp}^{-1}(T, H = 0) = 0 \tag{29}$$

$$\chi_{\parallel}^{-1}(T, H) = \frac{\partial H}{\partial M} = A + 3bM^2 = b(3M^2 - M_0^2) \tag{30}$$

$$\chi_{\parallel}^{-1}(T, H = 0) = 2bM_0^2. \tag{31}$$

The total spin-fluctuation (SF) contribution to the 'in-field' and 'zero-field' resistivity is the sum of the longitudinal and transverse contributions, i.e.,

$$\rho(T, H) = \rho_{\parallel}(T, H) + \rho_{\perp}(T, H)$$
$$= \frac{\pi^2}{9}\rho_0\gamma\left(\frac{k_B T}{\hbar\Gamma}\right)^2\left[\frac{\pi}{2}\sqrt{\frac{c}{b}}\frac{1}{\sqrt{3M^2 - M_0^2}} + \frac{2}{q_0}\left\{x\left(\frac{\pi}{2} - \tan^{-1}x + \frac{x}{1+x^2}\right)\right\}\right] \tag{32}$$

with

$$x = \frac{q_0}{\kappa_{\perp}} = q_0\sqrt{\frac{c}{b}}\frac{1}{\sqrt{M^2 - M_0^2}} \tag{33}$$

and

$$\rho(T, H = 0) = \rho_{\parallel}(T, H = 0) + \rho_{\perp}(T, H = 0) = \frac{\pi^2}{9}\rho_0\gamma\left(\frac{k_B T}{\hbar\Gamma}\right)^2\left[\frac{\pi}{2\sqrt{2}}\sqrt{\frac{c}{b}}\frac{1}{M_0} + \frac{2}{q_0}\right]. \tag{34}$$

In equations (32)–(34), we have set $c_{\parallel} = c_{\perp}$, $\gamma_{\parallel} = \gamma_{\perp}$ and hence $\Gamma_{\parallel} = \Gamma_{\perp}$. The negative magnetoresistance at *low* and *intermediate* temperatures is thus given by

$$\left(\frac{\Delta\rho}{\rho}\right)_{SF} = 1 - \frac{\rho(T, H)}{\rho(T, H = 0)} = 1 - \left[\frac{\pi}{2\sqrt{2}}\sqrt{\frac{c}{b}}M_0^{-1} + \frac{2}{q_0}\right]^{-1}$$
$$\times\left[\frac{\pi}{2}\sqrt{\frac{c}{b}}\frac{1}{\sqrt{3M^2 - M_0^2}} + \frac{2}{q_0}\left\{x\left(\frac{\pi}{2} - \tan^{-1}x + \frac{x}{1+x^2}\right)\right\}\right]. \tag{35}$$

In order to facilitate a direct comparison with the experiments, an attempt has been made to bring out the field dependence of the negative magnetoresistance clearly. For weak itinerant-electron ferromagnets, the quantity $(q_0/\kappa_{\perp})$ is *greater* than unity. Consequently, the function $\tan^{-1}(x)$ in equation (23) can be expanded as $\tan^{-1}(x) = (\pi/2) - (1/x) + (1/3x^3) - (1/5x^5) + (1/7x^7) - \cdots$. Retaining terms up to x^{-5} only in this expansion, equation (23) reduces to

$$\rho_{\perp}(T, H) = \rho_{\perp}(T, H = 0)\left[1 - \tfrac{2}{3}a_{\perp}H + \tfrac{3}{5}a_{\perp}^2H^2 - \cdots\right] \tag{36}$$

with

$$\rho_\perp(T, H=0) = \left(\frac{2\pi}{3}\right)^2 \gamma_\perp \left(\frac{\rho_0}{q_0}\right) \left(\frac{k_B T}{\hbar \Gamma_\perp}\right)^2 \tag{37}$$

and

$$a_\perp = \frac{1}{q_0^2 c_\perp M}. \tag{38}$$

For fields in the vicinity of $H = 0$, the magnetization at a given temperature can be expanded in a power series (Maclaurin's series) in H about $M(0)$:

$$M(H) = M(0) + HM'(0) + \frac{H^2}{2!}M''(0) + \frac{H^3}{3!}M'''(0) + \cdots. \tag{39}$$

Obtaining the first-, second- and third-order derivatives of magnetization with respect to field, evaluated at $H = 0$, i.e., $M'(0)$, $M''(0)$ and $M'''(0)$, from the MES, equation (25), and substituting in equation (39) yields the result

$$M(T, H) = M(T, H=0) - \left(\frac{1}{2A}\right) H + \frac{3}{8} M(T, H=0) \left(\frac{b}{A^3}\right) H^2 + \frac{1}{2}\left(\frac{b}{A^4}\right) H^3 - \cdots. \tag{40}$$

Recalling that $\chi_\parallel(T, H) \equiv \chi_\parallel(0) = \partial M(T, H)/\partial H$, $\chi_\parallel(T, H)$ can be calculated from equation (40) with the result

$$\chi_\parallel(T, H) = -\left(\frac{1}{2A}\right) + \frac{3}{4} M(T, H=0) \left(\frac{b}{A^3}\right) H + \frac{3}{2}\left(\frac{b}{A^4}\right) H^2 - \cdots.$$

Combining this result with the relation $\kappa_\parallel^2 = [c_\parallel \chi_\parallel(0)]^{-1}$ and equation (24), we obtain the longitudinal spin-fluctuation contribution to the 'in-field' resistivity as

$$\rho_\parallel(T, H) = \rho_\parallel(T, H=0)\left[1 - \tfrac{3}{2} a_\parallel H + \tfrac{39}{8} a_\parallel^2 H^2 - \cdots\right] \tag{41}$$

with

$$\rho_\parallel(T, H=0) = \frac{\pi^3}{18} \rho_0 \gamma_\parallel \left(\frac{c_\parallel}{-2A}\right)^{1/2} \left(\frac{k_B T}{\hbar \Gamma_\parallel}\right)^2 \tag{42}$$

and

$$a_\parallel = \frac{1}{2bM_0^3}. \tag{43}$$

Though the dependence of $\rho_\parallel(T, H)$ on field is explicit in equation (41), this dependence is valid only for fields close to $H = 0$. By contrast, the expression for $\rho_\parallel(T, H)$ in equation (32) holds over a wide range of fields, where the MES is obeyed. Thus the approach leading to equation (41) is abandoned and the earlier expression for $\rho_\parallel(T, H)$ is retained while the new expressions for $\rho_\perp(T, H)$ and $\rho_\perp(T, H=0)$, i.e., equations (36) and (37), replace the previous ones in equations (32) and (34). With this change, the negative magnetoresistance at *low* and *intermediate* temperatures assumes the form

$$\left(\frac{\Delta\rho}{\rho}\right)_{\mathrm{SF}} = 1 - \left[\frac{\pi}{2\sqrt{2}}\sqrt{\frac{c}{b}} M_0^{-1} + \frac{4}{q_0}\right]^{-1} \left[\frac{\pi}{2}\sqrt{\frac{c}{b}}\left(3M^2 - M_0^2\right)^{-1/2} + \frac{4}{q_0}\left(1 - \frac{2}{3}a_\perp H + \frac{3}{5}a_\perp^2 H^2\right)\right]. \tag{44}$$

In the derivation leading to equation (44), the lower cut-off wavevector q_0 and the longitudinal and transverse thermal variances $\langle m_\parallel^2 \rangle$ and $\langle m_\perp^2 \rangle$ are treated as field-independent quantities.

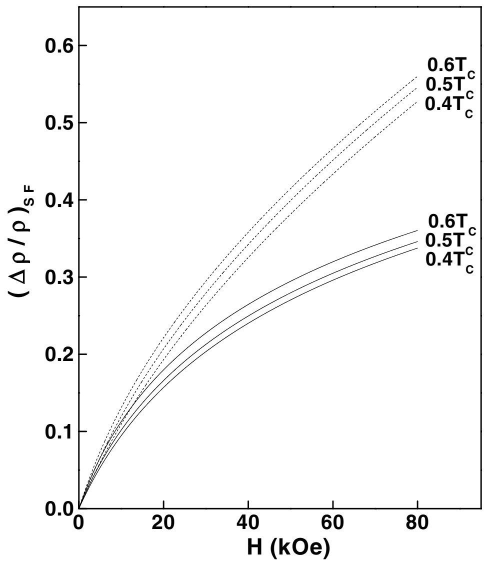

Figure 2. Variations of the spin-fluctuation contribution to negative magnetoresistance, $(\Delta\rho/\rho)_{SF}$, with magnetic field at $T = 0.4T_C$, $0.5T_C$ and $0.6T_C$ for Ni_3Al yielded by equation (35) (continuous curves) and (44) (dashed curves) when the reported [10, 13, 14] values of various physical parameters, appearing in these equations, are used.

Generally, this is not true; while q_0 increases slightly with H, $\langle m_{\parallel}^2\rangle$ and $\langle m_{\perp}^2\rangle$ get suppressed by H [13]. However, in the temperature range under consideration, the variations of these quantities with field are not appreciable and hence the assumption of neglecting their field dependences is not too drastic. Using the experimental values for various parameters, appearing in equations (35) and (44), for the weak itinerant-electron ferromagnet Ni_3Al from the literature [10, 13, 14], the field variations of $(\Delta\rho/\rho)_{SF}$ at fixed temperatures $0.4T_C$, $0.5T_C$ and $0.6T_C$ predicted for Ni_3Al by the expressions (35) and (44) are depicted in figure 2 by the continuous and dashed curves, respectively. The field variations of the negative magnetoresistance, so obtained, present the following salient features.

(i) At a fixed temperature, compared to the exact expression (35), the approximate expression (44) progressively overestimates the value of $(\Delta\rho/\rho)_{SF}$ with increasing field so much so that $(\Delta\rho/\rho)_{SF}$ increases steeply with no saturation in sight even at the highest field of 80 kOe. The theoretical field variations of $(\Delta\rho/\rho)_{SF}$, based on equation (35), are in better accord with the experiment [21, 22].

(ii) In agreement with the experimental observations [22], at a given field, $(\Delta\rho/\rho)_{SF}$ increases with temperature.

(iii) At *low* fields, the suppression of longitudinal and transverse spin fluctuations by the field, as reflected in $(\Delta\rho/\rho)_{SF}$, occurs in accordance with the power law $\sim H$ and the *linear* field dependence *dominates*over the *quadratic* one.

(iv) At *intermediate* and *not too high* fields, the term in equation (44), varying as $\sim H^2$, progressively *slows down* the *suppression* of both longitudinal and transverse spin fluctuations with increasing field so much so that $(\Delta\rho/\rho)_{SF}$ tends to saturate at high fields.

2.1.3. Temperatures around T_C but outside the critical region. The simplification resulting from the expansion of the trigamma function, appearing in equation (21), at moderate temperatures is not applicable at temperatures just outside the critical region but on either side of the Curie point, T_C (henceforth referred to as 'for temperatures close to T_C', for brevity), where such an expansion is no longer valid. However, equation (21) is amenable to an analytical solution if the so-called *classical approximation* is made. In this approximation, the spin-fluctuation cut-off wavevector q_c is chosen such that the spatially varying local magnetization, $m(r)$, is *classical*, which, in turn, implies that each local spin-density mode $m_\nu(q)$ for $q < q_c$ is thermally excited such that the part of the integrand in equation (2) involving the Bose function $n(\omega)$, i.e., $n(\omega)[n(\omega)+1]$ can be approximated by $(T/\omega)^2 - (1/12)$ for those values of ω for which Im $\chi_r(q,\omega)$ makes an appreciable contribution to the integral over ω in equation (2) and hence in equation (21). With this approximation, equation (21) can be cast into the form

$$r_\nu(T,h_r) = \left(\frac{2\gamma_\nu}{T}\right)\int_{q_\nu}^{q_c^\nu} \mathrm{d}q\, q^4 \left[T^2\int_0^\infty \frac{\mathrm{d}\omega}{\omega^2+\Gamma_\nu^2(q)} - \frac{1}{12}\int_0^\infty \frac{\omega^2\,\mathrm{d}\,\omega}{\omega^2+\Gamma_\nu^2(q)}\right] \tag{45}$$

where q_c^ν depends on both temperature and field [13]. At temperatures close to T_C, spin-fluctuation modes with $q \gg q_0$ significantly contribute to $r(T,h_r)$ so that the lower bound q_ν of the integral over q can be set equal to *zero* for *transverse spin fluctuations as well*, i.e., $q_\perp = q_\parallel = 0$. Making use of the reduced variable $x = q/q_c^\nu$, equation (45) reduces to

$$r_\nu(T,h_r) = \frac{\pi}{2}\gamma_\nu\left(\frac{T}{\Gamma_\nu}\right)(q_c^\nu)^2\left[1-y_\nu^2\left\{\ln(1+y_\nu^2)-\ln y_\nu^2\right\}\right] + \frac{\pi}{72}\gamma_\nu\left(\frac{\Gamma_\nu}{T}\right)(q_c^\nu)^8\left(\frac{3}{4}+y_\nu^2\right) - \frac{1}{30}\gamma_\nu(q_c^\nu)^5 \tag{46}$$

where $y_\nu = \kappa_\nu/q_c^\nu$.

The dependence of resistivity on magnetic field, implicit in the above expression, basically results from the variations of κ_ν and q_c^ν (and hence of y_ν) with field. The final expression for the spin-fluctuation contribution to the *negative* magnetoresistance, given below, is obtained by expressing the resistivity in normal units, employing the relations [13] $\kappa_\nu^2 = [c_\nu\chi_\nu(0)]^{-1}$ and

$$q_c^\nu(T,H) \cong \left(\frac{k_B T}{\hbar\Gamma_\nu}\right)^{1/3}(1-Z_\nu) \tag{47}$$

with

$$Z_\nu = \frac{1}{3c_\nu}\left(\frac{\hbar\Gamma_\nu}{k_B T}\right)^{2/3}\chi_\nu^{-1}(0) = \frac{1}{3}\left(\frac{\hbar\Gamma_\nu}{k_B T}\right)^{2/3}\kappa_\nu^2 \tag{48}$$

and taking cognizance of the fact that the expressions (28)–(31) for the 'zero-field' and 'in-field' susceptibilities for the longitudinal and transverse spin fluctuations are valid over a much wider field range spanning lower and lower fields as the temperature approaches T_C from either side (below or above). The expression in question is

$$\left(\frac{\Delta\rho}{\rho}\right)_{SF} = 1 - \frac{\rho_{SF}(T,H)}{\rho_{SF}(T,H=0)} \tag{49}$$

with

$$\rho_{SF}(T,H) = \frac{\pi}{6}\gamma\rho_0\left(\frac{k_B T}{\hbar\Gamma}\right)^{5/3}\Big[(1-Z_\parallel)^2[1-y_\parallel^2\{\ln(1+y_\parallel^2)-\ln y_\parallel^2\}] + 2(1-Z_\perp)^2[1-y_\perp^2\{\ln(1+y_\perp^2)-\ln y_\perp^2\}] - \frac{1}{15\pi}\left[(1-Z_\parallel)^5+2(1-Z_\perp)^5\right] + \tfrac{1}{36}\left\{(1-Z_\parallel)^8\left(\tfrac{3}{4}+y_\parallel^2\right)+2(1-Z_\perp)^8\left(\tfrac{3}{4}+y_\perp^2\right)\right\}\Big] \tag{50}$$

$$\rho_{\mathrm{SF}}(T,H=0)=\frac{\pi}{6}\gamma\rho_0\left(\frac{k_{\mathrm{B}}T}{\hbar\Gamma}\right)^{5/3}\left[(1-Z_{\|}^0)^2[1-y_{\|}^{0^2}\{\ln(1+y_{\|}^{0^2})-\ln y_{\|}^{0^2}\}]+2\right.$$
$$\left.-\frac{1}{15\pi}\left[(1-Z_{\|}^0)^5+2\right]+\frac{1}{36}\left\{(1-Z_{\|}^0)^8\left(\frac{3}{4}+y_{\|}^{0^2}\right)+\frac{3}{2}\right\}\right] \quad (51)$$

$$Z_{\|}=\frac{1}{3}\left(\frac{b}{c}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}\left(3M^2-M_0^2\right) \quad (52)$$

$$Z_{\|}^0=\frac{2}{3}\left(\frac{b}{c}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}M_0^2 \quad (53)$$

$$y_{\|}^2=\left(\frac{b}{c}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}(1-Z_{\|})^{-2}\left(3M^2-M_0^2\right) \quad (54)$$

$$Z_{\perp}=\frac{1}{3}\left(\frac{b}{c}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}\left(M^2-M_0^2\right)=\frac{1}{3c}\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}\left(\frac{H}{M}\right) \quad (55)$$

$$y_{\perp}^2=\left(\frac{b}{c}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}(1-Z_{\perp})^{-2}\left(M^2-M_0^2\right)=\frac{1}{c}\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3}(1-Z_{\perp})^{-2}\left(\frac{H}{M}\right) \quad (56)$$

$$Z_{\perp}^0=y_{\perp}^{0^2}=0. \quad (57)$$

In equations (50)–(57), $c_{\|}=c_{\perp}\equiv c$, $\gamma_{\|}=\gamma_{\perp}\equiv\gamma$ and hence $\Gamma_{\|}=\Gamma_{\perp}\equiv\Gamma$.

The field dependence of the negative magnetoresistance can be brought out explicitly as follows. Considering that $y<1$ at temperatures on either side of T_{C} but not very far from it and in external magnetic fields of moderate strength (note that $\chi_{\nu}^{-1}(0)=0$, and hence $y_{\nu}=0$, at $T=T_{\mathrm{C}}$ in the absence of an external magnetic field), the logarithmic functions appearing in equations (50) and (51) can be expanded in powers of y^2 with the result that the negative magnetoresistance still has the same form as equation (49) but with

$$\rho_{\mathrm{SF}}(T,H)=\frac{\pi}{2}\eta\rho_0\gamma\left(\frac{k_{\mathrm{B}}T}{\hbar\Gamma}\right)^{5/3}\left[1-\alpha\left\{b\left(3M^2-M_0^2\right)+2\left(\frac{H}{M}\right)\right\}\right.$$
$$\left.+\beta\left\{b^2\left(3M^2-M_0^2\right)^2+2\left(\frac{H}{M}\right)^2\right\}\right] \quad (58)$$

$$\rho_{\mathrm{SF}}(T,H=0)=\frac{\pi}{2}\eta\rho_0\gamma\left(\frac{k_{\mathrm{B}}T}{\hbar\Gamma}\right)^{5/3}\left[1-2\alpha bM_0^2+4\beta b^2M_0^4\right] \quad (59)$$

$$\eta=1-\frac{1}{15\pi}+\frac{1}{48} \quad (60)$$

$$\alpha=\frac{1}{27\eta c}\left(25-\frac{1}{\pi}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{2/3} \quad (61)$$

$$\beta=\frac{1}{81\eta c^2}\left(\frac{337}{4}-\frac{2}{\pi}\right)\left(\frac{\hbar\Gamma}{k_{\mathrm{B}}T}\right)^{4/3}. \quad (62)$$

Inserting the values of the parameters reported [10, 13, 14] for Ni_3Al in equations (49)–(62), equation (49) yields the variations of $(\Delta\rho/\rho)_{\mathrm{SF}}$ with the magnetic field at fixed temperatures $0.8T_{\mathrm{C}}$ and $0.9T_{\mathrm{C}}$ when either the expressions (50) and (51) (continuous curves in figure 3) or (58) and (59) (dashed curves in figure 3) are used for $\rho_{\mathrm{SF}}(T,H)$ and $\rho_{\mathrm{SF}}(T,H=0)$. Compared to the expressions (50) and (51), the approximate expressions (58) and (59) yield consistently lower values for $(\Delta\rho/\rho)_{\mathrm{SF}}$ at different fields at a given temperature but preserve the trend that, at a given field, $(\Delta\rho/\rho)_{\mathrm{SF}}$ increases with temperature. The theoretically predicted field variations at a given temperature as well as the temperature variations at a given value of

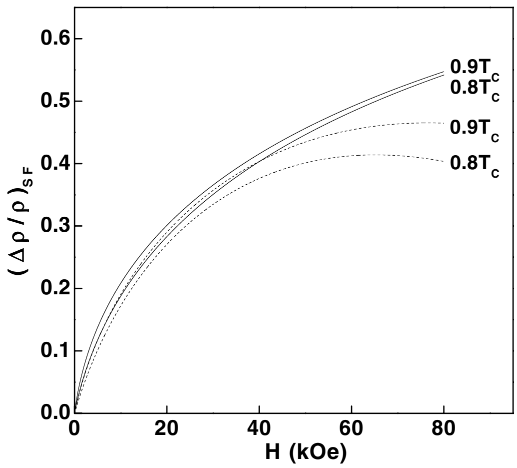

Figure 3. Negative magnetoresistance, due to thermally excited spin-density fluctuations, $(\Delta\rho/\rho)_{SF}$, as a function of magnetic field at $T = 0.8T_C$ and $0.9T_C$ for Ni_3Al predicted by equation (49) combined with either the expressions (50) and (51) (continuous curves) or (58) and (59) (dashed curves), when the reported [10, 13, 14] values of various physical parameters, appearing in these equations, are used.

H conform well with the experiments [21, 22]. From equation (44) and the expression (49) with $\rho_{SF}(T, H)$ and $\rho_{SF}(T, H = 0)$ given by equations (58) and (59), it is evident that, but for the difference in the actual structure of the coefficients of the H and H^2 terms, the field dependence of the spin-fluctuation contribution to magnetoresistance is of similar form for temperatures up to the Curie temperature.

3. Summary and conclusions

With a view to investigating the effect of magnetic field, H, on the low-lying magnetic excitations in weak itinerant-electron (WI) ferromagnets, the contributions to the electrical resistivity in the absence and presence of H, i.e., $\rho(T, H = 0)$ and $\rho(T, H)$, arising from the thermally excited *propagating transverse* spin fluctuations (spin waves) at low temperatures and *non-propagating* exchange-enhanced *longitudinal* and *transverse* spin-density fluctuations at intermediate temperatures and for temperatures close to the Curie point, T_C, have been calculated using a self-consistent approach that completely dispenses with the *unrealistic* electron gas approximation used in earlier theoretical treatments [1, 5, 6]. A close scrutiny of the expressions (15), (18), (20), (32)–(38), (44) and (49)–(62), so derived, reveals the following.

(i) The effect of magnetic field is to *leave* the *zero-field* functional form of the temperature dependence of resistivity *unaltered* but give rise to *field-dependent corrections* to the zero-field behaviour. Apart from a slight difference in the numerical factors, the expressions obtained in this work for the contributions to the zero-field resistivity, $\rho(T, H = 0)$, due to spin waves, $\rho_{SW}(T, H = 0)$, at low temperatures, equation (15), transverse and longitudinal spin-density fluctuations, $\rho_{\perp}(T, H = 0)$ and $\rho_{\parallel}(T, H = 0)$, at low and intermediate temperatures, equations (37) and (42), and spin-density fluctuations, $\rho_{SF}(T, H = 0)$, for temperatures close to T_C, equation (59), are the same as those derived earlier by Ueda and Moriya [5] based on the self-consistently renormalized (SCR) spin-fluctuation theory.

(ii) By contrast, the corresponding expressions for the *negative* magnetoresistance, i.e., equations (18)–(20), (36), (41), (44) and (59), have basically the same general form $(\Delta\rho/\rho) = 1 - [\rho(T, H)/\rho(T, H = 0)] \cong aH - bH^2$. Despite the similarity in the functional form, there are subtle distinctions in the way the field suppresses different types of magnetic excitations, as inferred from the magnetoresistance, primarily because the actual structure of the coefficients a and b depends on the nature of the spin-fluctuation modes (propagating or non-propagating, transverse or longitudinal) and the range of temperatures under consideration. This point is elucidated further in the following text.

In the expression for the spin-wave (SW) contribution, $(\Delta\rho/\rho)_{\rm SW}$, at low temperatures, equations (18) and (20), the coefficients a and b are *proportional to* $1/T$ and $1/T^2$, respectively, so that for a given field strength, the suppression of spin waves by the field *progressively weakens* with increasing temperature, whereas for a given temperature, it increases *linearly* with H at *low* fields but becomes *nonlinear* at *intermediate* and *high* fields where the H^2 term competes with the term linear in H. At intermediate temperatures, the coefficients a and b in equations (36), (41) and (44) have a negligibly weak dependence on temperature and field with the result that at *low* fields, the suppression of transverse and longitudinal spin fluctuations by the field follows the power law $\sim H$. However, at *intermediate* and *not too high* fields, according to equation (44), the *second* term in the expression for $(\Delta\rho/\rho)_\perp$, equation (36), varying as $\sim H^2$, progressively *slows down* the *suppression* of both transverse and longitudinal spin fluctuations with increasing field so much so that the total spin-fluctuation contribution, $(\Delta\rho/\rho)_{\rm SF}$, tends to saturate at high fields. At this stage, it is interesting to note that a fairly complicated expression for $(\Delta\rho/\rho)_{\rm SF}$ at low temperatures, derived by Ueda [6] using the SCR spin-fluctuation theory, also predicts a linear variation of $(\Delta\rho/\rho)_{\rm SF}$ with H at low fields but the field dependence at intermediate and high fields is not as obvious as in equation (44). For temperatures close to $T_{\rm C}$ also, the coefficients a and b in equation (58) are weakly dependent on temperature and field but *both the modes of spin fluctuations* get suppressed in accordance with the power law $\sim H$ at *low* fields and because of the presence of an additional term, varying as H^2, the total spin-fluctuation contribution to negative magnetoresistance tends to saturate at high fields.

The expressions for $(\Delta\rho/\rho)_{\rm SW}$ and $(\Delta\rho/\rho)_{\rm SF}$ obtained in this work are shown to permit a *quantitative* determination of the suppression of spin waves and spin-density fluctuations by the magnetic field in weakly ferromagnetic metals. Since the well-known physical quantities such as $g \cong 2$, $\mu_{\rm B}$ and $k_{\rm B}$ appear in the expression for $(\Delta\rho/\rho)_{\rm SW}$, equation (18) or (20), besides temperature and field, the theoretical T and H variations, predicted by equation (18) or (20) for the suppression of spin waves by field, are amenable to a *direct* comparison with the observed variations *without any free adjustable parameters*. By comparison, the coefficients of the H and H^2 terms in the expressions for $(\Delta\rho/\rho)_{\rm SF}$, i.e., in equation (44) and equation (49) combined with equations (58) and (59), involve the band parameters c_ν and γ_ν, which characterize the *static* and *dynamic* properties of $\chi_\nu(q,\omega)$, respectively. The parameters c_ν and γ_ν can be directly determined by measuring the Lorentzian linewidth $\Gamma_\nu(q) = \gamma_\nu q(\chi^{-1}(q) + c_\nu q^2)$ of $\mathrm{Im}\chi_\nu(q,\omega)/\omega$ as a function of q at different but fixed temperatures below and above $T_{\rm C}$ in neutron scattering experiments. Alternatively, a direct measurement of magnetization and spin-wave stiffness $D = g\mu_{\rm B}Mc_\perp$ (by inelastic neutron scattering) yields $c_\perp$. The values of the band parameters c_ν and γ_ν, so determined, are already available in the literature [1, 8, 10] for the archetypal weak itinerant-electron ferromagnets Ni_3Al, MnSi and $ZrZn_2$. Therefore, a quantitative comparison between theory and experiment for Ni_3Al, in particular, has been attempted by using the reported values of c_ν and γ_ν and the magnetization data [10, 13, 14] in the expressions (44) and (49)–(62) obtained in this work. A fairly good agreement between the theoretically predicted and experimentally observed [21, 22] temperature and field variations of the negative magnetoresistance has been found. In those cases where such a comparison is not possible because of the non-availability of the parameter values, c_ν and γ_ν can be treated as free fitting parameters while attempting a theoretical fit to the magnetoresistance data based on the present calculations. At this stage, it should be emphasized that the calculations presented in this work do not take into account the positive magnetoresistance contribution arising from the Lorentz force experienced by the conduction electrons due to the simultaneous presence of electrical and magnetic fields: this contribution is expected to become important particularly at low temperatures.

Acknowledgments

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Appendix

In this appendix, we make use of the classical approximation together with a temperature- and field-dependent spin-wave cut-off wavevector to derive analytical expressions for the suppression of the spin-wave contribution to electrical resistivity and magnetization by magnetic field for weak itinerant-electron ferromagnets.

In the *presence* of the magnetic field, the spin-wave cut-off wavevector q_0 *depends* on both the temperature (T) and the field (H). The variations of q_0 with T and H can be estimated from the condition

$$\omega(q_0) \cong T. \tag{A.1}$$

Combining equations (13) and (A.1) yields

$$q_0(T, h_r) = \pm[(T - h_r)/d_{SW}]^{1/2}. \tag{A.2}$$

In the *classical approximation*, the exponential term e^x in the Bose function $n(\omega) = 1/(e^{\omega(q)/T} - 1)$ is expanded in powers of x. If, in this expansion, the terms up to x^3 are considered so that the part of the integrand in equation (2) involving the Bose function, i.e., $n(\omega)[n(\omega) + 1]$ is approximated by

$$[T/\omega(q)]^2 - (1/12) + (13/12)[\omega(q)/T],$$

equation (14) reduces to

$$r(T, h_r) = \left(\frac{2\pi m_r T}{3d_{SW}}\right)\left[\int_0^{q_0} q\,dq - h_r \int_0^{q_0} \frac{q\,dq}{h_r + d_{SW}q^2}\right] - \left(\frac{\pi m_r}{18T}\right)\int_0^{q_0} q^3\omega(q)\,dq + \left(\frac{13\pi m_r}{18T^2}\right)\int_0^{q_0} q^3[\omega(q)]^2\,dq. \tag{A.3}$$

Using the value of q_0, given by equation (A.2), in the expression obtained after evaluating the above integrals and transforming back to *normal units*, leads finally to

$$\rho_{SW}(T, H) = \rho_{SW}(T, H = 0)[1 - 1.0615h - 0.0112h^3 + 0.0726h^4 + 0.8045h \ln h] \tag{A.4}$$

with

$$\rho_{\rm SW}(T, H = 0) = 0.4144\pi\rho_0 \left(\frac{g\mu_{\rm B}M}{\hbar}\right)\left(\frac{k_{\rm B}T}{D}\right)^2 \tag{A.5}$$

and the *reduced field* $h = (g\mu_{\rm B}H/k_{\rm B}T)$. Alternatively, the spin-wave contribution to the *negative* magnetoresistance at low temperatures is given by

$$\left(\frac{\Delta\rho}{\rho}\right)_{\rm SW} = 1 - \frac{\rho_{\rm SW}(T, H)}{\rho_{\rm SW}(T, H = 0)} = 1.0615h + 0.0112h^3 - 0.0726h^4 - 0.8045h\ln h. \tag{A.6}$$

This expression overestimates the absolute value of negative magnetoresistance obtained from equation (18) by roughly a factor of *two* and leads to a faster saturation, as is evidenced from figure 1, where the dotted and continuous curves depict the field variations predicted by equations (A.6) and (18), respectively.

If instead of retaining the terms up to x^3 in the expansion of the exponential term e^x, appearing in the Bose function $n(\omega) = 1/(e^{\omega(q)/T} - 1)$, the expansion is terminated at the second term only (i.e., $e^x \cong 1 + x$) and the calculations leading to equation (A.6) are repeated, we arrive at the result

$$\left(\frac{\Delta\rho}{\rho}\right)_{\rm SW} = \frac{2}{3}\left[-h\ln h + 2h - \frac{1}{2}h^2\right],$$

which has exactly the same form as equation (20) but overestimates the magnitude of $(\Delta\rho/\rho)_{\rm SW}$ by nearly a factor of *two*.

Next, the same theoretical formalism as above is used to calculate the spin-wave contribution to the magnetization at low temperatures. The thermal variances of local magnetization $\langle m_\parallel^2\rangle$ and $\langle m_\perp^2\rangle$ parallel ($\parallel$) and perpendicular ($\perp$) to the average magnetization, $\boldsymbol{M}$, are related to the imaginary part of the dynamical wavevector-dependent susceptibility, $\mathrm{Im}\chi_\nu(q,\omega)$, where ν is the polarization index $\parallel$ or $\perp$, through the well-known fluctuation-dissipation relation [1, 3, 10, 13]:

$$\langle m_\nu^2\rangle = \frac{8\hbar}{(2\pi)^3}\int_0^\infty q^2\mathrm{d}q \int_0^\infty \frac{\mathrm{d}\omega}{e^{\hbar\omega/k_{\rm B}T} - 1}\,\mathrm{Im}\chi_\nu(q,\omega). \tag{A.7}$$

Using the expression for $\mathrm{Im}\chi_\perp(q,\omega)$, given by equation (11), and the spin-wave dispersion relation, equation (12), equation (A.7) can be put into the form

$$\langle m_\perp^2\rangle = \frac{2g\mu_{\rm B}M}{(2\pi)^2}\int_0^\infty \frac{q^2\,\mathrm{d}q}{e^{\hbar\omega(q)/k_{\rm B}T} - 1}. \tag{A.8}$$

When $H = 0$, $\hbar\omega(q) = Dq^2$ and the integral over q in equation (A.8) yields

$$\langle m_\perp^2\rangle = \zeta(3/2)g\mu_{\rm B}M(T,0)\left(\frac{k_{\rm B}T}{4\pi D}\right)^{3/2}. \tag{A.9}$$

With the aid of the approximate form of the magnetic equation of state in the *absence* of the magnetic field [13], i.e., $M(T,0)/M(0,0) \cong 1 - \langle m_\perp^2\rangle/M^2(0,0)$, the expression for the spin-wave contribution to spontaneous magnetization, equation (A.9), can be cast into the well-known Bloch form

$$M(T,0) = M(0,0) - \zeta(3/2)g\mu_{\rm B}\left(\frac{k_{\rm B}T}{4\pi D}\right)^{3/2}. \tag{A.10}$$

In the *presence* of an external magnetic field, equation (A.8) assumes the form

$$\langle m_\perp^2\rangle = \frac{g\mu_{\rm B}M}{(2\pi)^2}\left(\frac{k_{\rm B}T}{D}\right)^{3/2}\int_0^\infty \frac{x^{1/2}\mathrm{d}x}{e^{x+h} - 1},$$

where $x = Dq^2/k_BT$ and $h = g\mu_B H/k_BT$. The integral can be solved exactly to yield the final result

$$M(T,H) = M(0,0) - g\mu_B\left(\frac{k_BT}{4\pi D}\right)^{3/2}\zeta(3/2,h)$$
$$= M(0,0) - g\mu_B\left(\frac{k_BT}{4\pi D}\right)^{3/2}[\zeta(3/2) - 3.54h^{1/2} + 1.64h - 0.104h^2 + 0.004\,25h^3] \tag{A.11}$$

with $\zeta(3/2) = 2.612$.

We now make use of the *classical* approximation, i.e., approximate the Bose function $[e^{\hbar\omega(q)/k_BT} - 1]^{-1}$ by $[k_BT/\hbar\omega(q)] - (1/2) + (1/12)[\hbar\omega(q)/k_BT]$ and replace the upper limit (∞) of the integral over q by the spin-wave cut-off wavevector q_0, which depends on the temperature and field in accordance with the relation

$$q_0(T,H) = q_0(T,H=0)\,[1-h]^{1/2}, \tag{A.12}$$

where $q_0(T,H=0) = (k_BT/D)^{1/2}$. Note that equation (A.2), when expressed in usual units, is nothing but equation (A.12). In the classical approximation, equation (A.8) assumes the form

$$\langle m_\perp^2\rangle = \frac{g\mu_B M(T,H)}{2\pi^2}\left[\int_0^{q_0}\frac{k_BTq^2\,dq}{g\mu_BH + Dq^2} - \frac{1}{2}\int_0^{q_0}q^2\,dq + \frac{1}{12}\int_0^{q_0}q^2\left(\frac{\hbar\omega(q)}{k_BT}\right)dq\right]$$
$$= \frac{g\mu_B M(T,H)}{2\pi^2}\left(\frac{k_BT}{D}\right)\left[q_0 - \left(\frac{g\mu_BH}{D}\right)^{1/2}\tan^{-1}\left\{q_0\left(\frac{g\mu_BH}{D}\right)^{-1/2}\right\}\right]$$
$$+ \frac{g\mu_B M(T,H)}{12\pi^2}\left(\frac{k_BT}{D}\right)^{3/2}(1-h)^{3/2}\left[\frac{h}{6} - 1 + \frac{1}{10}(1-h)\right]. \tag{A.13}$$

At weak and moderate fields and for typical values of other quantities appearing in the argument x of the function $\tan^{-1}(x)$, $x > 1$. Consequently, $\tan^{-1}(x)$ can be expanded as $\tan^{-1}(x) \cong (\pi/2) - (1/x) + (1/3x^3) - (1/5x^5) + \cdots$ and equation (A.13) finally yields the result

$$\langle m_\perp^2\rangle \cong \frac{4}{\pi^{1/2}}g\mu_B M(T,H)\left(\frac{k_BT}{4\pi D}\right)^{3/2}\left[\frac{51}{60} - \frac{\pi}{2}h^{1/2} + \frac{53}{72}h - \frac{1}{32}h^2 + \frac{7}{960}h^3\right]. \tag{A.14}$$

At low temperatures and moderate fields, the magnetic equation of state [13]

$$\left[\frac{M(T,H)}{M(0,0)}\right]^2 = 1 - \left(\frac{T}{T_C^S}\right)^2 - \frac{3\langle m_\parallel^2\rangle + 2\langle m_\perp^2\rangle}{M^2(0,0)} + 2\chi(0,0)\frac{H}{M(T,H)}, \tag{A.15}$$

where T_C^S is the Stoner Curie temperature (which is usually extremely high) and $\chi(0,0)$ is the zero-field differential susceptibility at 0 K (typically $\approx 10^{-3}$ for weak itinerant-electron ferromagnets), can be approximated by the expression

$$\frac{M(T,H)}{M(0,0)} \cong 1 - \frac{\langle m_\perp^2\rangle}{M^2(0,0)}. \tag{A.16}$$

Combining equations (A.14) and (A.16), we finally obtain

$$M(T,H) \cong M(0,0) - g\mu_B\left(\frac{k_BT}{4\pi D}\right)^{3/2}[1.918 - 3.545h^{1/2} + 1.661h - 0.071h^2 + 0.0165h^3]. \tag{A.17}$$

A comparison between equations (A.11) and (A.17) asserts that the expression (A.17), which makes use of the classical approximation as well as the temperature- and field-dependent spin-wave cut-off wavevector, provides a reasonably accurate description of the suppression of the spin-wave contribution to the magnetization by magnetic field in weak itinerant-electron ferromagnets. In conformity with the experimental observations [14, 16, 19] made on the weakly ferromagnetic metallic alloys $Ni_{75}Al_{25}$ and $Ni_{75-x}Fe_xAl_{25}$, the expression (A.17) yields the $H^{1/2}$ law for the suppression of spin waves by low and moderate fields in weak itinerant-electron ferromagnets, as reflected in the spin-wave contribution to the magnetization.

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